Six-roll sugarcane mill: mathematical model, finite element analysis and its validation by strain-gauge measurement

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Abstract Each type of mill, depending on the number and configuration of rolls and type of drive, requires a separate mathematical model. Published work on mathematical models of mills have largely ignored the friction power absorbed at the roller journals. The force distribution among various mill components has been computed or assumed, but there has been very little validation of stress/strain values through field measurements. This paper reports on an opportunity for field measurement of total torque and its distribution between different rollers, development of a reliable mathematical model, solving of the model, static structural analysis, and its validation through strain-gauge measurement. A new mathematical model was developed for a six-roll mill and used for static structural analysis to obtain its stress pattern. Input values for the mathematical model, such as friction power at the roller journals and distribution of total torque between different rolls, were collected from laboratory tests and operating data of the milling tandem, respectively. Real-time strain-gauge measurement was carried out on a 1980 mm long six-roll penultimate mill of the 8000 t cane/day milling tandem at Saraswati Sugar Mills, India during the 2015-16 harvest, in collaboration with the Automotive Research Association of India. Strain gauges were installed at 32 locations on the mill and its grooved pressure feeder. A portable data-acquisition system was deployed to acquire time-series data from the strain gauges and the DCS recorded mill-operating parameters. The acquired data was processed and analysed to determine the principal strain at different locations of the mill assembly. These strain values were converted into stress values using Young's modulus and then compared with the analytical values to fine-tune the model.

Key words Mathematical model, six-roll mill, FEA, strain gauge measurement

INTRODUCTION

In this paper, a mathematical model is defined as a set of mathematical equations, derived from resolution of various forces acting on a component. The derived equations predict the final pattern of forces. There are more than a dozen types of sugarcane mills in operation worldwide. However, the most popular are four-roll and six-roll mills. Each type of mill, depending on the number and configuration of rolls and the type of drive, requires a separate mathematical model.

Patil (1999), developed a mathematical model of a four-roll mill but it ignored the frictional forces at the roller journal. This model was not validated through any field measurement. Here, we develop a mathematical model of a six-roll mill, with the model being validated through field measurements using strain gauges in order to establish a robust and reliable system for optimizing mill designs. We validated the model on a six-roll mill at Saraswati Sugar Mills, India, a cane-sugar factory with a milling tandem of 8000 t cane/day capacity. A typical six-roll mill (Fig. 1) comprises an under feed roller mounted on a grooved roller pressure feeder (GRPF) connected to a three-roll mill through a pressure chute.

This paper describes the five steps of the study: development of a mathematical model, solving of the model, finite element analysis (FEA), real-time strain-gauge measurements, and finally comparison of field data with FEA results for a 1980 mm long six-roll mill. The operating parameters of the mill were inputs for the mathematical model that gave output in the form of various reaction forces. These forces were, in turn, used as inputs to the static model of the mill to generate the stress pattern, which was then compared with field measured gauge data.

DEVELOPMENT OF A NEW MATHEMATICAL MODEL OF A SIX-ROLL MILL

A 3D model of the mill under study is shown in Figure 1. The main three-roll mill is driven by a variable speed electric motor and foot-mounted reduction gearing connected through a tail bar to the top roller. A set of crown pinions drive the bottom rollers. The GRPF is driven by a hydraulic motor mounted on its bottom roller shaft with a torque arm. A pair of crown pinions drive the GRPF top roller, which in turn drives the under feed roller through another pair of crown pinions.
Table 1 gives the symbols we use.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumed power at mill drive</td>
<td>$P_1$</td>
<td>kW</td>
</tr>
<tr>
<td>Consumed power at GRPF drive</td>
<td>$P_2$</td>
<td>kW</td>
</tr>
<tr>
<td>Mill roller speed</td>
<td>$N_1$</td>
<td>r/min</td>
</tr>
<tr>
<td>GRPF roller speed</td>
<td>$N_2$</td>
<td>r/min</td>
</tr>
<tr>
<td>Torque</td>
<td>$T$</td>
<td>kN-m</td>
</tr>
<tr>
<td>Force</td>
<td>$F$</td>
<td>kN</td>
</tr>
<tr>
<td>Normal force transmission ratio, discharge/ feed</td>
<td>$r_c$</td>
<td></td>
</tr>
<tr>
<td>Torque transmission ratio, discharge/ feed roll</td>
<td>$r_b$</td>
<td></td>
</tr>
<tr>
<td>Friction dissipation factor at trash plate</td>
<td>$\beta$</td>
<td></td>
</tr>
<tr>
<td>Trash plate contact angle with feed roller</td>
<td>$\alpha$</td>
<td></td>
</tr>
<tr>
<td>Angle of inclination of GRPF with vertical</td>
<td>$\beta$</td>
<td></td>
</tr>
<tr>
<td>Fraction of GRPF torque transmitted to UFR</td>
<td>$\varphi$</td>
<td></td>
</tr>
<tr>
<td>Angle of action of normal force exerted by UFR on top roller</td>
<td>$\lambda$</td>
<td></td>
</tr>
<tr>
<td>Coefficient of friction at roller journal</td>
<td>$\mu$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 gives the nomenclature for force and torque. For ease of understanding, the symbols are kept the same as used by Patil (1999), since this study is an extension of his work. Additional terms are used for the new parameters, i.e. friction torque at the roller journals, forces acting on the pressure chute, and GRPF components.
Table 2. Nomenclature for force and torque.

<table>
<thead>
<tr>
<th>No</th>
<th>Description</th>
<th>Mill Rollers</th>
<th>GRPF Rollers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top Roll</td>
<td>Feed Roll</td>
</tr>
<tr>
<td>1</td>
<td>Friction torque at roller bearing</td>
<td>Tg₁</td>
<td>Tg₁</td>
</tr>
<tr>
<td>2</td>
<td>Torque at roller shell</td>
<td>Tc₁</td>
<td>Tc₁</td>
</tr>
<tr>
<td>3</td>
<td>Friction torque at trash plate</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>4</td>
<td>Crown pinion force (tangential)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>5</td>
<td>Crown pinion force (radial)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>6</td>
<td>Opposing crushing force (tangential)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>7</td>
<td>Crushing force (radial)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>8</td>
<td>Trash plate opposing force (tangential)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>9</td>
<td>Trash plate crushing force (normal)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>Torque arm reaction on GRPF Roller</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>11</td>
<td>Friction couple on mill top bearing housing</td>
<td>R₁f</td>
<td>...</td>
</tr>
<tr>
<td>12</td>
<td>Couple on mill top bearing housing due to top cap eccentricity</td>
<td>R₁e</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Vertical reaction on bearing housing (pintle/drive)</td>
<td>R₁v₁ / R₁v₂ / R₁v₃ /</td>
<td>R₁g₁v₁ / R₁g₁v₂ / R₁g₁v₃ /</td>
</tr>
<tr>
<td>14</td>
<td>Horizontal reaction on bearing housing (pintle/drive)</td>
<td>R₁h₁ / R₁h₂ / R₁h₃ /</td>
<td>R₁g₁h₁ / R₁g₁h₂ / R₁g₁h₃ /</td>
</tr>
</tbody>
</table>

**Torque distribution on mill rollers**

We assumed that all the torque consumed by the mill bottom rollers is delivered by the top roll through the mill crown pinions only. A similar assumption was made for the GRPF rollers.

The input torque delivered at the mill top roller square end is distributed between the mill top roller, the mill feed roller, the mill discharge roller and the trash plate as shown below:

\[ \text{Total torque at mill top roll} = (T_{tc} + T_{tf} + T_{tr}) + (T_{fc} + T_{ff}) + (T_{dc} + T_{df}) \]  
(1)

The input torque delivered at the GRPF bottom roller drive end is distributed between the GRPF bottom roller, the GRPF top roller and the under feed roller as shown below:

\[ \text{Total torque at PF bottom roll} = (T_{gbf} + T_{gbc}) + (T_{gbf} + T_{gbc}) + (T_{gbf} + T_{gbc}) \]  
(2)

**Typical formulae for calculation of forces on various components**

Figure 2 shows a diagram of the hydraulic force distribution on the mill rollers:

\[ \text{Crushing force radial due to cane on feed roller shell} \ (F_{nc}) = \frac{(\text{Total hyd load} - F_{trc})}{(1 + r_c) \cdot \cos \frac{\phi}{2}} \]  
(3)

where \( r_c = \frac{F_{trc}}{F_{nc}} \)  
(4)

Figure 3 shows a diagram of the crown pinion forces:

\[ \text{Tangential force on pinion teeth} \ (F_{fp}, F_{dp}, F_{gfp}, F_{gup}) = \frac{\text{Torque consumed by respective roller shell} + \text{friction torque at bearing}}{\text{respective pinion PCB/2}} \]  
(5)

\[ F_{fnp} = F_{bp} \cdot \tan \phi \]  
(6)
Friction Torque at bearing = Reaction force at bearings, \( \mu \cdot \frac{d}{2} \) (7)

\[ F_{fc} = \frac{\tau_{fc} - \tau_{ff}}{\text{(respective roller shell nom. dia}/2)} \] (8)

\[ F_{tr} = \frac{\tau_{tr}}{\text{Distance from top roller centre to trash plate top}} \] (9)

\[ F_{TA} = \frac{\text{Torque at GRPF}}{\text{Torque arm length}} \] (10)

**Fig. 2.** Distribution of hydraulic load on the mill rollers.

**Fig. 3.** Crown pinion loads.

**Free body diagrams for rollers**

To solve the engineering problem involving a polygon of forces, a free body diagram of each mill roll was prepared as shown in Figures 4-9. The reactions at the roller bearing housing are calculated from equations associated with the respective roller.

The free body diagram for the mill top roller is given in Figure 4.
Fig. 4. Forces acting on the mill top roller.

The top roller equations for the bearing reactions are:

\[ \begin{align*}
R_{t1v} + \mu R_{t1h} &= -R_{t2v} + (F_{fn} + F_{fp}) \cos \frac{\alpha}{2} + F_{trc} \cos \beta + (F_{fp} + F_{fc}) \sin \frac{\alpha}{2} + (F_{dn} + F_{dc}) \cos \frac{\alpha}{2} - (F_{dp} + F_{dc}) \sin \frac{\alpha}{2} \\
&+ F_{trc} \sin \beta - \mu R_{t2h} \quad (11) \\
R_{t1h} - \mu R_{t1v} &= -R_{t2h} - (F_{fp} + F_{fc}) \cos \frac{\alpha}{2} - F_{trc} \cos \beta + (F_{fn} + F_{fp}) \sin \frac{\alpha}{2} - (F_{dn} + F_{dc}) \sin \frac{\alpha}{2} - (F_{dp} + F_{dc}) \cos \frac{\alpha}{2} \\
&+ F_{trc} \sin \beta + \mu R_{t2v} \quad (12)
\end{align*} \]

\( R_{t2v} \) and \( R_{t2h} \) can be derived from the moment equations:

\[ \begin{align*}
R_{t2v} + \mu R_{t2h} &= \frac{1}{L} \left[ -\frac{L}{2} \left( -F_{fn} \cos \frac{\alpha}{2} - F_{fc} \sin \frac{\alpha}{2} - F_{trc} \sin \beta - F_{dn} \cos \frac{\alpha}{2} - F_{trc} \cos \beta + F_{dc} \sin \frac{\alpha}{2} \right) \\
&- (L + K) \left( -F_{fn} \cos \frac{\alpha}{2} - F_{fp} \sin \frac{\alpha}{2} - F_{dn} \cos \frac{\alpha}{2} + F_{dp} \sin \frac{\alpha}{2} \right) \right] \quad (13)
\]

\[ \begin{align*}
R_{t2h} - \mu R_{t2v} &= \frac{1}{L} \left[ -\frac{L}{2} \left( -F_{fn} \sin \frac{\alpha}{2} + F_{fc} \cos \frac{\alpha}{2} + F_{trc} \cos \beta + F_{dn} \sin \frac{\alpha}{2} + F_{dc} \cos \frac{\alpha}{2} - F_{trc} \sin \beta \right) \\
&- (L + K) \left( -F_{fn} \sin \frac{\alpha}{2} + F_{fp} \cos \frac{\alpha}{2} + F_{trc} \sin \beta + F_{dn} \sin \frac{\alpha}{2} + F_{dp} \cos \frac{\alpha}{2} \right) \right] \quad (14)
\]

The free body diagram for the mill discharge roller is given in Figure 5.
Fig. 5. Forces acting on the mill discharge roller.

The discharge roller equations for the bearing reactions are:

\[ R_{d1v} + \mu R_{d1h} = -R_{d2v} - \mu R_{d2h} + (F_{dnp} + F_{dnc}) \cos \frac{\alpha}{2} + F_{dc} \sin \frac{\alpha}{2} - F_{dp} \sin \frac{\alpha}{2} \]  
(15)

\[ R_{d1h} - \mu R_{d1v} = -R_{d2h} + \mu R_{d2v} + (F_{dnc} + F_{dnp}) \sin \frac{\alpha}{2} + F_{dp} \cos \frac{\alpha}{2} - F_{dc} \cos \frac{\alpha}{2} \]  
(16)

\( R_{d1v} \) and \( R_{d1h} \) can be derived from the moment equations:

\[ R_{d2v} + \mu R_{d2h} = \frac{1}{L} \left[ \frac{L}{2} (F_{dnc} \cos \frac{\alpha}{2} + F_{dc} \sin \frac{\alpha}{2}) + (L + K) \left( F_{dnp} \cos \frac{\alpha}{2} - F_{dp} \sin \frac{\alpha}{2} \right) \right] \]  
(17)

\[-R_{d2h} + \mu R_{d2v} = \frac{1}{L} \left[ \frac{L}{2} \left( -F_{dnc} \sin \frac{\alpha}{2} + F_{dc} \cos \frac{\alpha}{2} \right) - (L + K) \left( F_{dnp} \sin \frac{\alpha}{2} + F_{dp} \cos \frac{\alpha}{2} \right) \right] \]  
(18)

The free body diagram for the mill feed roller is given in Figure 6.
The feed roller equations for the bearing reactions are:

$$R_{f1v} - \mu R_{f1h} = -R_{f2v} + \mu R_{f2h} + (F_{fnp} + F_{fnc}) \cos \frac{\alpha}{2} + F_{fp} \sin \frac{\alpha}{2} - F_{fc} \sin \frac{\alpha}{2} \quad (19)$$

$$R_{f1h} + \mu R_{f1v} = -R_{f2h} - \mu R_{f2v} + (F_{fnc} + F_{fnp}) \sin \frac{\alpha}{2} + F_{fc} \cos \frac{\alpha}{2} - F_{fp} \cos \frac{\alpha}{2} \quad (20)$$

$R_{f2v}$ and $R_{f2h}$ can be derived from the moment equations:

$$-R_{f2v} + \mu R_{f2h} = \frac{1}{L} \left[ -\frac{L}{2} \left( F_{fnc} \cos \frac{\alpha}{2} - F_{fc} \sin \frac{\alpha}{2} \right) - (L + K) \left( F_{fnp} \cos \frac{\alpha}{2} + F_{fp} \sin \frac{\alpha}{2} \right) \right] \quad (21)$$

$$-R_{f2h} - \mu R_{f2v} = \frac{1}{L} \left[ -\frac{L}{2} \left( F_{fnc} \sin \frac{\alpha}{2} + F_{fc} \cos \frac{\alpha}{2} \right) - (L + K) \left( F_{fnp} \sin \frac{\alpha}{2} - F_{fp} \cos \frac{\alpha}{2} \right) \right] \quad (22)$$

The free body diagram for the GRPF bottom roller is given in Figure 7.

**Fig. 6.** Forces acting on the mill feed roller.
The GRPF bottom roller equations for the bearing reactions are:

\[ R_{gb2v} - \mu R_{gb2h} = -R_{gb1v} + \mu R_{gb1h} + F_{gnc} + F_{gnp} - F_{TA} \cos \theta \quad (23) \]

\[ R_{gb2h} + \mu R_{gb2v} = -R_{gb1h} - \mu R_{gb1v} + F_{gc} + F_{gp} + F_{TA} \sin \theta \quad (24) \]

\[ R_{gb1v} \text{ and } R_{gb1h} \text{ can be derived from the moment equations:} \]

\[ R_{gb1v} - \mu R_{gb1h} = \frac{1}{L} \left[ \frac{L}{2} F_{gnc} + (L + K) F_{gnp} - (L + K') F_{TA} \cos \theta \right] \quad (25) \]

\[ R_{gb1h} + \mu R_{gb1v} = \frac{1}{L} \left[ \frac{L}{2} F_{gc} + (L + K) F_{gp} + (L + K') F_{TA} \sin \theta \right] \quad (26) \]

The free body diagram for the GRPF top roller is given in Figure 8.
Fig. 8. Forces acting on the GRPF top roller.

The GRPF top roller equations for the bearing reactions are:

\[ R_{gt2v} - \mu R_{gt2h} = -R_{gt1v} + \mu R_{gt1h} + F_{gmp} + F_{gnc} + ((L + K)/(L + K')) L T \cos \theta + F_{gup} \cos \phi + F_{guc} \cos \phi + F_{gunc} \sin \phi + F_{gump} \sin \phi \]  

(27)

\[ R_{gt2h} + \mu R_{gt2v} = -R_{gt1h} - \mu R_{gt1v} + F_{gc} - F_{gpp} + F_{gup} \sin \phi + F_{guc} \sin \phi - F_{gunc} \cos \phi - F_{gump} \cos \phi \]  

(28)

\[ R_{gt1v} - \mu R_{gt1h} = \frac{1}{L} \left[ (L + K) (F_{gmp}) + \frac{L}{2} (F_{gnc} + F_{guc} \cos \phi + F_{gunc} \sin \phi) + ((L + K') L T \cos \theta + K' (-F_{gump} \sin \phi - F_{gup} \cos \phi) \right] \]  

(29)

\[ R_{gt1h} + \mu R_{gt1v} = \frac{1}{L} \left[ -(L + K) (F_{gpp}) + \frac{1}{2} (F_{gc} + F_{guc} \sin \phi - F_{gunc} \cos \phi) + K' (-F_{gup} \sin \phi + F_{gump} \cos \phi) \right] \]  

(30)

The free body diagram for the under feed roller is given in Figure 9.
**Fig. 9.** Forces acting on the under feed roller.

The under feed roller equations for the bearing reactions are:

\[-R_{gu2v} + \mu R_{gu2h} = R_{gu1v} - \mu R_{gu1h} - F_{gup} \cos \phi + F_{guc} \cos \phi + F_{gun} \sin \phi + F_{gup} \sin \phi \quad (31)\]

\[-R_{gu2h} - \mu R_{gu2v} = R_{gu1h} + \mu R_{gu1v} - F_{gup} \sin \phi + F_{guc} \sin \phi - F_{gun} \cos \phi - F_{gup} \cos \phi \quad (32)\]

\[R_{gu1v} \text{ and } R_{gu1h} \text{ can be derived from the moment equations:}\]

\[-R_{gu1v} + \mu R_{gu1h} = \frac{1}{L} \left\{ \frac{L}{2} \left( F_{guc} \cos \phi + F_{gun} \sin \phi \right) + K' \left( F_{gup} \cos \phi - F_{gun} \sin \phi \right) \right\} \quad (33)\]

\[R_{gu1h} + \mu R_{gu1v} = \frac{1}{L} \left\{ \frac{L}{2} \left( F_{gunc} \cos \phi - F_{guc} \sin \phi \right) - K' \left( F_{gump} \cos \phi + F_{gup} \sin \phi \right) \right\} \quad (34)\]

**Diagrams for couple forces on the bearing housing**

The free body diagram for the mill top roller bearing housing is given in Figure 10.
The couple forces on the bearing housing due to eccentricity are:

\[ R_c = \frac{R_v \cdot e - \text{(Resultant bearing reaction)} \cdot \text{(Journal radius)} \cdot \sin \phi}{\text{square root (bearing width}^2 + \text{bearing height}^2)} \]  \hspace{1cm} (35)

The couple forces on the bearing housing due to friction are:

\[ R_{cf} = \frac{(\text{Resultant bearing reaction)} \cdot \text{(Journal radius)} \cdot \mu}{\text{square root (bearing width}^2 + \text{bearing height}^2)} \]  \hspace{1cm} (36)

The resultant bearing reaction is the sum of the roller reaction and liner friction. Direction of couple \( R_c \) is dependent on the extent of eccentricity.

The free body diagrams for the GRPF bearing housing are given in Figure 11.

The movement of bagasse through the chute between the GRPF and the mill causes a force on the mill headstock in the direction of bagasse travel, which causes a reaction force on the GRPF rollers in the opposite direction (Fig. 12).
Solving the mathematical model

Table 3 gives the operating parameters for model solving.

**Table 3.** Details and operating parameters of the six-roll, 1980 mm long mill.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Unit</th>
<th>three-roll Mill</th>
<th>GRPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roller nominal diameter</td>
<td>D</td>
<td>mm</td>
<td>915</td>
<td>915</td>
</tr>
<tr>
<td>Operating crush rate</td>
<td>t/h</td>
<td></td>
<td>330</td>
<td></td>
</tr>
<tr>
<td>Installed drive rating</td>
<td>kW</td>
<td></td>
<td>650 (DC motor)</td>
<td>315</td>
</tr>
<tr>
<td>Consumed drive power</td>
<td>P1, P2</td>
<td>kW</td>
<td>365</td>
<td>130</td>
</tr>
<tr>
<td>Operating roller speed</td>
<td>N1, N2</td>
<td>r/min</td>
<td>5.4</td>
<td>6.3</td>
</tr>
<tr>
<td>Crown wheel PCD</td>
<td>A</td>
<td>mm</td>
<td>992</td>
<td></td>
</tr>
<tr>
<td>Roll journal diameter</td>
<td>d</td>
<td>mm</td>
<td>460</td>
<td></td>
</tr>
<tr>
<td>Roller journal centers</td>
<td>L</td>
<td>mm</td>
<td>3100</td>
<td></td>
</tr>
<tr>
<td>Top bearing housing</td>
<td>wxh</td>
<td>mm</td>
<td>530 x 325</td>
<td></td>
</tr>
<tr>
<td>Hydraulic load on top roll (per unit length)</td>
<td>--</td>
<td>t/m</td>
<td>206</td>
<td>--</td>
</tr>
</tbody>
</table>

Kharbanda and Pandey (2013) carried out laboratory tests to determine the friction coefficient between the mill roller journal and the gun metal bearing. This was extrapolated for white metal lined bearings installed at the mill under study.

Goel et al. (2015) estimated the torque distribution between the mill and GRPF of a six-roll mill with separate drives for GRPF and the mill, and Lewinski et al. (2013) estimated the torque distribution between the top, feed and discharge rolls of a four-roll mill. Table 4 gives the estimated values of torque and force distribution used for the FEA.

**Table 4.** Distribution of force and torque.

<table>
<thead>
<tr>
<th>Description</th>
<th>Source</th>
<th>Symbol</th>
<th>3 roll mill</th>
<th>GRPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff. of friction at roll journals in white metal lined bearings</td>
<td>Field data</td>
<td>μ</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>% torque distribution between mill rolls (feed/discharge/top)</td>
<td>Field data</td>
<td></td>
<td>23.75 / 52.5</td>
<td></td>
</tr>
<tr>
<td>% torque distribution between GRPF rolls (top/bottom/UFR)</td>
<td>Assumed</td>
<td></td>
<td>46 / 46 / 8</td>
<td></td>
</tr>
<tr>
<td>Force ratio for mill: discharge/feed roll</td>
<td>Field data</td>
<td>r_c</td>
<td>7:1</td>
<td></td>
</tr>
<tr>
<td>Force ratio for the GRPF: top/bottom roller</td>
<td>Assumed</td>
<td></td>
<td>1:1</td>
<td></td>
</tr>
<tr>
<td>Push force in pressure chute</td>
<td>Assumed</td>
<td>F_p</td>
<td>171 kN</td>
<td></td>
</tr>
<tr>
<td>Fraction of hyd. force on trash plate</td>
<td>Field data</td>
<td>b</td>
<td>0.2 of total hydraulic load</td>
<td></td>
</tr>
</tbody>
</table>
The vertical and horizontal reaction forces on the roller bearing housings have been calculated at mill operating parameters, using equations 11-34 and are summarised in Table 5. A value prefixed by the sign (\(\cdot\)) indicates the reaction force is opposite to the direction of arrow in the force diagram (Fig. 13).

**Table 5.** Vertical and horizontal reaction forces on bearing housing (kN).

<table>
<thead>
<tr>
<th>Description</th>
<th>Mill roller bearing housing</th>
<th>GRPF roller bearing housing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top roll</td>
<td>Feed roll</td>
</tr>
<tr>
<td>Drive side housing</td>
<td>(R_{h2})</td>
<td>1468</td>
</tr>
<tr>
<td></td>
<td>(R_{v2})</td>
<td>2255</td>
</tr>
<tr>
<td>Pintle side housing</td>
<td>(R_{h1})</td>
<td>847</td>
</tr>
<tr>
<td></td>
<td>(R_{v1})</td>
<td>2060</td>
</tr>
</tbody>
</table>

Table 6 shows the reaction forces of the trash plate and the bearing couple, calculated using equations 9 and 35-37. The pressure chute forces were extrapolated using field data.

**Table 6.** Forces of trash plate, pressure chute and bearing couple (kN).

<table>
<thead>
<tr>
<th>Description</th>
<th>Trash plate</th>
<th>Pressure chute</th>
<th>Mill top bearing housing</th>
<th>GRPF bearing housings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ftr</td>
<td>Ftrc</td>
<td>Fgc</td>
<td>Rc</td>
</tr>
<tr>
<td>Mill drive side</td>
<td>25</td>
<td>401</td>
<td>171</td>
<td>446</td>
</tr>
<tr>
<td>Mill pintle side</td>
<td>25</td>
<td>401</td>
<td>216</td>
<td>49</td>
</tr>
</tbody>
</table>

**Fig. 13.** Typical force diagram of the six-roll mill assembly.
Static structural FEA of the six-roll mill assembly

The calculated reaction forces on various mill components given in Tables 5 and 6 were used as the input for the static structural FEA of the assembly using ANSYS workbench software version R15. The model is fixed at the foundation bolts with the base resting on rigid floor.

FEA results in the form of a Von-Mises stress pattern is shown in Figure 14.

![Von Mises stress pattern for the mill assembly](image)

**Fig. 14.** Von Mises stress pattern for the mill assembly.

Real-time strain-gauge measurement and its analysis

The strain-gauge measurement was carried by ARAI (Automotive Research Association of India). In all, 32 locations were identified as shown in Figures 15-17. The gauges were pasted in position and suitably protected a week prior to the start of the 2015 harvest season.
Fig. 15. Strain gauge locations with numbering.

Fig. 16. Numbering system for the gauges.

Fig. 17. Gauge installed at the mill headstock.
After stabilization of the crushing operations, data was acquired using an 80-channel eDAQ portable system made by Somat (USA). It continued for 3 consecutive days: 10 hours on 19 December, 10 hours on 20 December and 8 hours on 21 December 2015. Operating data, such as roller speed, load on the mill and GRPF drives, top roller lift, hydraulic pressure and cane crushing rate, were also collected.

The received data was filtered before presentation for analysis. A typical pattern of time series filtered data is presented in Figures 18 and 20 for the side cap and foundation zone of the headstock, respectively. Figure 19 shows the strain pattern of the rosette gauge for the feed side cap at the start of the mill. Figure 21 shows an out-of-phase curve of mill load versus the strain (location 2 of Figure 15) at the headstock.

**Fig. 18.** Filtered data for the rosette strain gauge mounted at the side cap at feed side, location mark 7 (Fig. 15).

**Fig. 19.** Strain pattern at the start of the mill for the gauge mounted at the side cap at the feed side.

**Fig. 20.** Filtered data for the rosette strain gauge mounted at the head stock, location mark 1 (Fig. 15).

**Fig. 21.** Time series comparison of mill load versus gauge mounted at head stock, location mark 2 (Fig. 15).

**Comparison of field data with the FEA result**

The filtered field data was received from ARAI in the form of strain in microns for each of the 32 locations. Young’s modulus, a ratio of stress and strain, having a value of 210,000 MPa for carbon steel was used to convert the RMS strain values into stress values and then compared with the values generated by FEA. The mathematical model was then corrected and the final FEA results are given in Table 7.

The comparison between the stress values derived from the final FEA model and the RMS values based on field measurements over 3 days is summarised in Table 7. The negative values denote compression, while the positive values denote tension.

**Observations**

During the course of validation of the mathematical model, initial assumptions were corrected based on the strain gauge measured data to obtain the final FEA stress values. Key observations of the study are:

- Discharge versus feed roller hydraulic force transfer ratio ($r_c$) has been updated from the initial assumed value of 2 to the field result, which in this case is 7.
The unit loading on the GRPF rollers has been corrected from the initial assumed value of 80 t/m to the field result, which in this case is 26 t/m.

The frictional power loss at mill and GRPF bearings with white metal lining is approximately 20% and 10%, respectively, of the consumed power. The high frictional power loss in the mill is due to the high constant hydraulic pressure, while in the GRPF the top cap pre-tensioning screw pressure has a fixed lower value.

The forces on the two sides of the headstocks (drive and pintle) are quite different, particularly at the foundation bolts and side cap on the feed side.

The GRPF headstock tends to pull the mill headstock towards itself, whereas the pressure chute pushes the mill headstock towards the discharge side.

The FEA has limitations with regard to loads on foundation bolts, with the field measured data differing widely from the FEA results.

### Table 7. Comparison of field measured values with theoretical stress values for six-roll mill.

<table>
<thead>
<tr>
<th>Gauge mark no./type (Figure 15)</th>
<th>Stress value (MPa)</th>
<th>Gauge mark no./type (Figure 15)</th>
<th>Stress value (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From field measurement</td>
<td>Calculated from FEA model</td>
<td>From field measurement</td>
<td>Calculated from FEA model</td>
</tr>
<tr>
<td>Gauge fixed at headstock side face</td>
<td></td>
<td>Gauge fixed at GRPF side face</td>
<td></td>
</tr>
<tr>
<td>01 R(+)</td>
<td>34.4</td>
<td>12 R(+/−)</td>
<td>7.9</td>
</tr>
<tr>
<td>02 R(+/−)</td>
<td>14.3</td>
<td>13 U (+)</td>
<td>2.9</td>
</tr>
<tr>
<td>03 R(+)</td>
<td>8.4</td>
<td>14 R (+)</td>
<td>3.9</td>
</tr>
<tr>
<td>04 R(+/−)</td>
<td>5.4</td>
<td>Gauge fixed at GRPF top cap</td>
<td></td>
</tr>
<tr>
<td>05 R (+)</td>
<td>7.3</td>
<td>15 U (−)</td>
<td>4.6</td>
</tr>
<tr>
<td>06 R(+/−)</td>
<td>9.4</td>
<td>Gauge fixed at turn beam</td>
<td></td>
</tr>
<tr>
<td>07 R (+)</td>
<td>3.7</td>
<td>16A Bad data</td>
<td></td>
</tr>
<tr>
<td>08 R (+/−)</td>
<td>8.1</td>
<td>16B U (+)</td>
<td>11.1</td>
</tr>
<tr>
<td>Gauge fixed at side caps</td>
<td></td>
<td>Gauge fixed at scraper levers</td>
<td></td>
</tr>
<tr>
<td>09 R(+/−)</td>
<td>8.1</td>
<td>17 FB</td>
<td>Bad data</td>
</tr>
<tr>
<td>09 R(+/−)</td>
<td>8.1</td>
<td>18 U</td>
<td>Bad data</td>
</tr>
<tr>
<td>09 R(+/−)</td>
<td>8.1</td>
<td>19 FB</td>
<td>Bad data</td>
</tr>
<tr>
<td>09 R(+/−)</td>
<td>8.1</td>
<td>Gauge fixed at UFR bearing housing</td>
<td></td>
</tr>
<tr>
<td>10 R(+)</td>
<td>1.8</td>
<td>20 R</td>
<td>Bad data</td>
</tr>
<tr>
<td>11 R(+)</td>
<td>15.9</td>
<td>Gauge fixed at pressure chute</td>
<td></td>
</tr>
<tr>
<td>11 R(+)</td>
<td>15.9</td>
<td>21 R</td>
<td>Bad data</td>
</tr>
<tr>
<td>11 R(+)</td>
<td>15.9</td>
<td>Gauge fixed at GRPF stools</td>
<td></td>
</tr>
<tr>
<td>11 R(+)</td>
<td>15.9</td>
<td>22 R</td>
<td>Bad data</td>
</tr>
<tr>
<td>11 R(+)</td>
<td>15.9</td>
<td>Gauge fixed at GRPF stools</td>
<td></td>
</tr>
<tr>
<td>12 R(+/−)</td>
<td>2.1</td>
<td>24A U(−)</td>
<td>3.4</td>
</tr>
<tr>
<td>12 R(+/−)</td>
<td>2.1</td>
<td>24B U(−)</td>
<td>1.2</td>
</tr>
<tr>
<td>13 U(−)</td>
<td>1.2</td>
<td>24C U(−)</td>
<td>2.6</td>
</tr>
<tr>
<td>13 U(−)</td>
<td>1.2</td>
<td>24D U(−)</td>
<td>2.5</td>
</tr>
<tr>
<td>14 R(+)</td>
<td>17.2</td>
<td>(+) Tension, (+/−) the three arms of rosette gauges are elongated or compressed, (−) compression</td>
<td></td>
</tr>
<tr>
<td>14 R(+)</td>
<td>17.2</td>
<td>R: Rosette type strain gauge, U: Uni-axial type strain gauge, FB: Full bridge type strain gauge</td>
<td></td>
</tr>
<tr>
<td>14 R(+)</td>
<td>17.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 R(+)</td>
<td>17.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### CONCLUSIONS

We have developed a new mathematical model of a six-roll mill and then solved it for a 1980 mm size mill operating at 330 t/h, after assuming certain ratios for force and torque distribution. The calculated reaction forces were used as input for the real-time finite element model. FEA results were then obtained and compared with the strain-gauge field data to fine-tune the model and the various ratios. The validated model is used for optimising the mill design.

### ACKNOWLEDGMENT

We thank the Saraswati Sugar Mill Ltd, Yamunanagar (India) and ARAI, Pune for their support in carrying out strain-gauge measurement.
REFERENCES


Moulin à 6 rouleaux: modèle mathématique, analyse par éléments finis et validation par mesure de contrainte

Résumé. Chaque type de moulin, selon le nombre et la configuration des rouleaux et le type de conduite, nécessite un modèle mathématique particulier. Les travaux publiés sur les modèles mathématiques des moulins ont largement ignoré l'énergie absorbée par les coussinets. La distribution de force entre les différents composants du moulin a été calculée ou supposée, mais il y a eu très peu de validation des valeurs de contrainte par des mesures sur le moulin. Cet article examine des mesures sur un moulin du couple total et de sa répartition entre différents rouleaux, et propose un modèle mathématique fiable, son application, ses résultats et sa validation par des mesures de contraintes. Un nouveau modèle mathématique a été développé pour un moulin de 6 rouleaux et utilisé pour l'analyse statique et pour le stress. Les valeurs d'entrées pour le modèle mathématique, comme l'énergie de friction dans les coussinets et la répartition du couple total entre les différents rouleaux, ont été obtenues par des essais de laboratoire et des mesures faites sur les moulins de la sucrerie, respectivement. Les mesures de la jauge de contrainte en temps réel ont été réalisées dans l'avant-dernier moulin du tandem, avec 6 rouleaux de 1980 mm, broyant 8000 t canne/jour à Saraswati, lors de la campagne de 2015-16, en collaboration avec l'Automotive Research Association of India. Les jauge de contraintes ont été installées à 32 emplacements sur le moulin et sur son « pressure feeder ». Un système d'acquisition de données a été déployé pour acquérir les données chronologiques. Les données acquises ont été analysées afin de déterminer la contrainte principale à différents endroits de l'ensemble du moulin. Ces valeurs ont été converties en valeurs de stress et ensuite comparées avec les valeurs analytiques pour améliorer le modèle.

Mots-clés: Modèle mathématique, mesures, moulin de 6 rouleaux, FEA, jauge de contrainte

Molino azucarero de 6 mazas: modelo matemático, analisis de elemento finito y su validacion con mediciones de tensometro

Resumen. Cada tipo de molino, dependiendo del numero y configuracion de las mazas y el tipo de transmision, requiere un modelo matematico separado. Los trabajos publicados sobre los modelos matematicos de los molinos han casi siempre ignorado el poder de la friccion absorvido en los muñones de las mazas. La distribucion de fuerza entre varios componentes del molino ha sido calculado o asumido, pero ha habido poca validacion de los valores de stress/tension a traves de las mediciones de campo. Este trabajo reporta sobre la oportunidad de las mediciones de campo del torque total y su distribucion entre diferentes mazas, desarrollo de un modelo matematico confiable, la resolucion del modelo, analisis estatico estructural y su validacion a traves de mediciones con tensometro. Un nuevo modelo matematico fue desarrollado para un molino de 6 mazas y utilizado para analisis estatico estructural para obtener su patron de stress. Valores de entrada para el modelo matematico, como la fuerza de friccion en los muñones de la maza y la distribucitn del torque total entre las diferentes mazas, fueron recolectadas de las pruebas de laboratorio y de la informacion de operacion del tandem de mazos, respectivamente. Mediciones con tensometro en tiempo real fueron realizadas en el penultimo molino de 6 mazas de 1980 mm en un tandem con molienda diaria de 8000 tcd en el ingenio Saraswati sugar, india, durante la zafras 2015/16, en colaboracion con la Asociacion de Investigacion Automotriz de India. Los tensometros fueron instalados en 32 puntos en el molino y en el alimintador forzado rayado. Un sistema portatil para recepcion de informacion fue desplegado para recibir informacion en series de tiempo de los tensometros y los parametros de operacion de molino registrados en el dcs. La informacion recibida fue procesada y analizada para determinar la tension principal en las diferentes posiciones del molino. Estos valores de las tensiones fueron convertidos en valores de tension usando el young modulus y luego comparado con los valores de analisis del modelo afinado.

Palabras clave: Modelo matematico, molino de 6 mazas, fea, mediciones de tensometro